160218

**The choices for your project.**

**Tell me your project choice by March 4.**

**If you work in a group on a project, no two of you in the group may choose the same *OPTIONS.***

***If more than one person tackles Project 7 I encourage you to work together.***

Project choice 1:

As discussed in our first two classes global climate is warming, however the temperature change at given location may be quite different than the global average. Furthermore even if the average temperature over many years (climate timescale) at a given location warms that does not mean that a given day of the year becomes consistently warmer. There is variability in warming from year to year, month to month and day to day, and definitely on these timescales we often see cooling. What I failed to mention in class is my concern that climate variability, in the case of temperature how much ups and downs there are, may be systematically increasing, a possible serious danger to ecosystems and humanity.

The standard deviation of a set of data values, {di}, is the square root of the variance, where the variance is given by <(di - <di>)2>, with the angle brackets, <>, representing the mean. The covariance of two sets of data values, {d1i} and {d2i}, is given by <(d1i - <d1i>)(d2i - <d2i>)>. The ratio of the covariance of the data sets to the product of their standard deviations, <(d1i - <d1i>)(d2i - <d2i>)>)/(<(d1i - <d1i>)2>1/2 <(d2i - <d2i>)2>1/2 ), is the correlation coefficient which measures to what extent the two data sets vary together. It ranges in value from 1 (perfect correlation, they vary the same way) through 0 (no correlation, they vary in independent ways) to -1 (perfect anti-correlation, they vary oppositely).

Project:

1. Find data for the temperature in New York City for each day of February in 1976 (when I was a teenager) and 2016. Use *OPTION 1: a data set all of whose values are for the same time of day (and state the time)* ***OR*** *OPTION 2: a data set of highs for the day* ***OR*** *OPTION 3: a data set of lows for the day* ***OR*** *OPTION 4: a data set of means for the day.* Make plots vs. day of month for temperature for both years’ Februarys and of the difference between the temperatures on corresponding dates of the two years.
2. Calculate the mean of each year’s data set and the difference of the means.
3. Calculate the standard deviations of the data sets for both years.
4. Calculate the correlation coefficient for the two years.
5. Comment on whether your results suggest that New York City may be warming or cooling, or whether the climate variability in New York City may be increasing or decreasing, or whether the patterns of temperature within the month may have changed in New York City.

Project choice 2:

The heat and salt diffusivities, KH and KS (minus the ratios of heat flux and salt flux respectively to temperature and salinity gradient respectively) measures of the tendency of heat and salt respectively to mix, generally decrease with the Richardson number Ri (the ratio of Brunt Vaisala frequency squared to shear squared), a measure of the stratification. This is because stratification suppresses turbulence and turbulence is the main cause of mixing. For a stably stratified case, Ri>0, the difference between KS and KH tends to increase as the salt-heat stability parameter Rρapproaches 1. Rρ is minus the ratio of the salt contribution to the potential density gradient divided by the heat contribution to the potential density gradient, and when it approaches one in stable stratification the stabilizing effect of one of heat or salt is approaching being undone by the destabilizing effect of the other driving a strong small-scale “double diffusive” instability.

I am using data from the Goddard Institute Space Studies (GISS) model E ocean to evaluate ocean vertical mixing parameterizations that the turbulence group at GISS, of which I am a part, is developing. These parameterizations are part of the GISS Vertical Mixing Scheme (GISSVM) which is intended to provide more realistic vertical mixing for climate models.

Project:

1. Write a program that calculates the properly weighted means and standard deviations of Ri,Rρ,KH,KS and the correlation coefficients of Ri and KH and Rρ and (KS/KH) on a north-south section of ocean data from GISS model E that I will give you.
2. Run your program and present the results. Comment on how the results on the correlation coefficients compare with expectations.
3. Make either *OPTION 1: scatterplots* ***OR*** *OPTION 2: 2-D histograms* to show the distribution of data points in Ri,KH space, Ri,KS space & Rρ,(KS/KH) space.
4. Based on what you see in step 2 suggest a way to restrict the range(s) of one or more of the variables that you expect to lead to a value of at least one of the correlation coefficients closer to expectations than in part 1.
5. Modify your program to calculate statistics with the restricted range of variables you proposed in part 3.
6. Repeat steps 2 and 3 with your modified program
7. Comment on your results.

Project choice 3:

In your textbook “Introduction to Computational Science” by Shiflet&Shiflet, module 2.2 discusses unconstrained growth, which is simply exponential growth, the opposite of exponential decay, and module 2.3 discusses constrained growth.

In the Euler forward method, which I will call “forward scheme”, the average value of the time derivative of a function over a time interval from t to t+Δt is approximated using input values from time t. In the Euler backward method, which I will call “backward scheme”, it is approximated using input values from time t+Δt. A third scheme, uses for the input values the mean of those at time t and those at time t+Δt, I will call this “forward and backward scheme”.

Project:

1. Write programs to simulate dP/dt = 0.05 P using
2. the “forward scheme”
3. the “backward scheme”
4. the “forward and backward scheme”
5. Run each of your 3 programs from t=0 to t=100 and graph population, P(t), as well as the proportional error, (P(t)-Pexact(t))/Pexact(t). Where Pexact is the exact solution of the differential equation dP/dt = 0.05P. If the absolute value of the maximum proportional error is not less than 1% for any of your programs then find and state a timestep that makes it less than 1% . Run that(those) programs over with corrected timestep and repeat the graphs of the population and the proportional error.
6. State how your timestep compares with the number 1/0.05 . State which of the schemes a), b) or c) is best for this problem.
7. Using *either OPTION1: The scheme you found best for the unconstrained growth problem in step3* ***OR*** *OPTION2: a scheme found in Chapter 6 of your book which is different from each of a), b) and c),* write a difference equation and program, analogous to the ones you used in part 1, that approximates instead dP/dt = 0.05P – 0.003P2 (as seen in exercise 5 in Module 2.3 of the textbook).
8. Run it from t=0 to t=100 and graph population, P(t) in this case.

(I know no exact solution here so don’t do proportional error.)

Project choice 4:

The pendulum is discussed in module 3.3 of your textbook. A linear pendulum model is an approximation to a real pendulum adequate for small angle displacements than admits of an analytic solution. A nonlinear pendulum model must be used for large angles. Linear drag is an approximation to air resistance good for slow motion. If motion becomes fast drag is nonlinear. The larger the maximum angle displacement the greater the maximum speed and the more likely that nonlinear drag must be considered. Nonlinear drag formulas can be complicated but the sum of a linear and a quadratic drag term is a plausible approximation for moderate speeds.

With g being the acceleration of gravity, l the length of the pendulum, m the mass of the pendulum bob and θ the angle the pendulum makes with the vertical:

The evolution equation for the angle in a linear pendulum model without drag is:

A. d2θ/dt2 = -(g/l) θ .

For a linear pendulum model with linear drag the evolution equation is:

B. d2θ/dt2 = -(g/l) θ – (k1/(l m)) dθ/dt , where k1 is a coefficient for Stokes (linear) air friction.

For a nonlinear pendulum model without drag the equation is:

C. d2θ/dt2 = -(g/l) sin(θ) .

For a nonlinear pendulum model with linear drag it is:

D. d2θ/dt2 = -(g/l) sin(θ) – (k1/(l m)) dθ/dt .

For a nonlinear pendulum model with a nonlinear drag composed of linear and quadratic terms:

E. d2θ/dt2 =-(g/l) sin(θ)–(k1/(l m))dθ/dt –(k2/(l m))(dθ/dt)2 [k2=coefficient for quadratic friction].

Project:

1. Write programs to simulate *either OPTION 1: equations A,B and C* ***OR*** *OPTION 2: equations B and D* ***OR*** *OPTION 3: equations B and E*.
2. Choose values of m,g,l and ,if necessary, k1, k2 .
3. Choose a timestep and explain why it is reasonable to think your timestep will give a reasonably accurate solution.
4. Run each progam with initial condition d/dt=0 and =0.1 and graph (t) for at least five complete swings back and forth.
5. Run each program with initial condition d/dt=0 and =/2 and graph (t) for the same period of time as the run in part 4.
6. Run each program with initial condition d/dt=2(g/r)1/2 and =0 and graph (t) for the same period of time as the run in part 4.

Project choice 5:

Module 3.1 of your book says a little about falling motion.

If z is the height, the equation of motion for an object moving vertically through a fluid (a gas such as air or a liquid such as water) under the influence of gravity and the resistance of the fluid is d2z/dt2 = -g – (Fdrag/m)(dz/zt)/|dz/dt|) , where m is the mass, |dz/dt| is the speed, and Fdrag is the magnitude of the resistance force of the fluid. Fdrag= k1 ||**v**|| with k a constant for Stokes friction, an approximation appropriate for slow motion, and Fdrag = k2 v2 for Newtonian friction, with k2 a constant, an approximation for moderately great speed. If speed varies from slow to fast using the sum of Stokes & Newtonian friction, Fdrag= k1 ||**v**|| + k2 v2 may be better.

Project:

1. Write a difference equation to describe an object moving vertically under the influence of gravity alone, i.e. the motion if it were in a vacuum.
2. Write a program to implement this difference equation.
3. Take g to be 9.8 m/s2 and run with initial conditions z=0 anddz/dt= a) 10.0 m/s AND b) dz/dt=-10.0m/s. Graph z(t) from t=0 to t=4s for BOTH.
4. Write a difference equation to describe an object moving vertically through a fluid under gravity and the resistance of the fluid where the resistance of the fluid is given by Stokes friction.
5. Write a program to implement this difference equation
6. Choose an m and a k1 and run this program with the same initial conditions as in 3 and graph z(t) from t=0 to t=4s.
7. Write a difference equation to describe an object moving vertically through a fluid under gravity and the resistance of the fluid where the resistance is given by *OPTION 1: Newtonian friction* ***OR*** *OPTION 2: sum of Newtonian and Stokes friction*. Repeat steps 5 and 6 for this difference equation.

Project choice 6:

If z is the height and x is the horizontal position, the equations of motion of a projectile moving under the influence of gravity alone are d2z/dt2 = -g and d2x/dt2 = 0 . These are two separate differential equations to be solved separately and the separate solutions are then combined to give the position as a function of time: (x(t),z(t)) . These equations apply to an object moving through a vacuum near the surface of a planet over a region that is small compared to the planet.

If a projectile instead of moving through a vacuum is moving slowly through a fluid, we can approximate the viscosity, a.k.a. the resistance of the fluid, by Stokes friction with some constant k as in project choice 3. The differential equations describing the motion become: d2z/dt2 = -g – (k/m)dz/dt and d2x/dt2 = -(k1/m)dx/dt . Because the drag is linear the two equations are still completely separate. If the projectile is shot rapidly through a fluid, the problem calls for a nonlinear drag formula, such as Fdrag = k2 v2 for Newtonian friction as in project choice 5 , the two differential equations for the z and x components of the position then become coupled by the drag term:, since v2 = ((dx/dt)2 +(dz/dt)2) involves both x and z components of the velocity.

Project:

1. Write the formula for the analytic solution of the motion of a projectile moving under gravity in terms of the time, t, initial position, (x0,z0), initial speed, v0, and the angle, 0, the initial direction the velocity is above the horizontal, and write the formula for the analytic solution of the x component of position when the projectile reaches height 0 (the range).
2. Write difference equations to describe a projectile moving under the influence of gravity alone.
3. Write a program to implement these difference equations and calculate the x and z components of position and their differences from the analytic solution as functions of time.
4. Choose values of x0, z0, & a slow value of v0 and an angle between 150 and 75o. Graph x vs. t, y vs. t & their differences from the analytic solution vs. t.
5. Write the difference equation to describe a projectile moving under gravity and a viscosity given by *either OPTION 1: Newtonian friction* ***OR*** *OPTION 2: Stokes friction* & implement it in your program.
6. Choose a k2 and run this program with the same initial conditions as in 4 and graph x vs.t, y vs. t and make a plot in 2 dimensions of the path [which is the curve parameterized by (x(t),y(t))].

Project choice 7:

For a satellite such as CUNYSAT spearheaded by Dr. Shermane Austin, in simplest approximation equations of motion are d2x/dt2 = -G MEarth x /(x2 + y2)3/2  and

d2y/dt2 = -G MEarth y /(x2 + y2)3/2, where x and y are the components of position in a plane whose origin is at the center of the Earth, MEarth is the mass of the Earth and G is the Universal Gravitational Constant. This approximation assumes gravity is the only force acting on the space-craft, that the only body whose gravity matters is the Earth and that the Earth has perfect spherical symmetry: all idealizations. In the special case that the initial position is x=R, y=0 and the initial velocity dx/dt=0 ,dy/dt = (GMEarth/R)1/2 ,which gives an orbital period ~ ½ our classtime for a very near-Earth orbit, the analytic solution to these simplest equations is (x,y)(t)=(cos(t)R,sin(t)R), where =(GMEarth/R3)1/2 . Their general solution is a Keplerian path.

If we add just the linear static approximation to the gravitational perturbation due to a masspoint at the Moon’s distance, dMoon, with the Moon, fixed on the +x axis (the actual Moon orbits but only through a small angle in half a CS 345 classtime), we get (MMoon=Moon’s mass):

d2x/dt2 = -G MEarth x /(x2 + y2)3/2 + (2GMMoon/dMoon3)x and d2y/dt2 = -G MEarth y /(x2 + y2) 3

Project:

1. Mark Richardson worked on calculating a satellite orbit in this class in 2014. Obtain his code for the simplest approximation.
2. Take the initial conditions to be x(0)=R, y(0)=0 and dx/dt(0)=0, dy/dt(0) = (GMEarth/R)1/2 and run the program for 48 model hours.
3. Plot the path [the curve parameterized by (x(t),y(t))] for the analytic solution and from your simulation. Comment on how they compare.
4. Write the difference equations to describe the motion of an Earth satellite including the Moon’s effect in the linear static approximation.
5. Modify the program to implement these difference equations and calculate the position as a function of time (x(t),y(t)).
6. Take the initial conditions to be x(0)=R, y(0)=0 and dx/dt(0)=0, dy/dt(0) = (GMEarth/R)1/2 and run your program for 48 model hours.
7. Plot the path from the results of your simulation. Comment on how it compares to the plots in 3.
8. Repeat steps 6 and 7 for the different initial conditions *EITHER OPTION 1: x(0)=R, y(0)=0 and dx/dt(0)=0, dy/dt(0) = (1.5 GM/R)1/2* ***OR*** *OPTION 2: initial conditions* x(0)=R, y(0)=0 and dx/dt(0)=0, dy/dt(0) = (2 GM/R)1/2 .

Project choice 8 for the Resourceful:

GISS has a version of the atmospheric climate model designed especially for student use, the EdGCM. Unfortunately the EdGCM has compatibility problems with Linux that have stymied me in the past. In 2014 a couple students managed to run EdGCM.

This project is for you to research what they did and get it running for you. Then run for 100 years climate simulation with current conditions and *either OPTION 1: Carbon dioxide concentration constant 1.5 times the present concentration* ***OR*** *OPTION 2: carbon dioxide concentration progressively increasing over the 100 years from the present concentration to twice the present concentration.*

Then calculate

1)the means of the temperatures over the whole atmospheres at the ends of the two simulations and the difference of the means.

2) the standard deviations of the temperatures over the whole atmospheres at the ends of the simulations.

3) the standard deviation of the difference of the temperatures over the whole atmospheres at the ends of the simulations.

4) the correlation coefficient of the temperatures in the two simulations over the whole atmospheres.

5) Repeat steps 1) through 4) for just the temperatures at the surface.